Identity – a statement that is true for all values for the variables given

Trigonometric identity – an identity using trigonometry expressions

**Example 1**
Prove that \( \sec x \cot x = \sin x \) is *not* a trigonometric identity by producing a counterexample.

Suppose \( x = \frac{\pi}{3} \).

\[
\begin{align*}
\sec x \cot x & \neq \sin x \\
\sec \frac{\pi}{3} \cot \frac{\pi}{3} & \neq \sin \frac{\pi}{3} \quad \text{Replace } x \text{ with } \frac{\pi}{3}.
\end{align*}
\]

Reciprocal Identities
Sin \( \Theta \) = 1/csc \( \Theta \) and csc \( \Theta \) = 1/sin \( \Theta \)
Cos \( \Theta \) = 1/sec \( \Theta \) and sec \( \Theta \) = 1/cos \( \Theta \)
Tan \( \Theta \) = 1/cot \( \Theta \) and cot \( \Theta \) = 1/tan \( \Theta \)

Quotient Identities
sin \( \Theta \)/cos \( \Theta \) = tan \( \Theta \) and cos \( \Theta \)/sin \( \Theta \) = cot \( \Theta \)

Using the unit circle, and \( x^2 + y^2 = 1 \) then \((\cos \Theta)^2 + (\sin \Theta)^2 = 1\) so,

**Example 2**
Use the given information to find the trigonometric value.

a. If \( \cot \theta = \frac{6}{5} \), find tan \( \theta \).

\[
\begin{align*}
tan \theta & = \frac{1}{\cot \theta} \\
& = \frac{1}{\frac{6}{5}} \quad \text{Choose an identity that involves tan } \theta \text{ and cot } \theta. \\
& = \frac{5}{6} \quad \text{Substitute } \frac{6}{5} \text{ for cot } \theta \text{ and evaluate.}
\end{align*}
\]
b. If \( \sec \theta = \frac{5}{4} \), find \( \cot \theta \).

Since there are no identities relating \( \sec \theta \) and \( \cot \theta \), we must use two identities, one relating \( \sec \theta \) and \( \tan \theta \) and another relating \( \cot \theta \) and \( \tan \theta \).

\[
\sec^2 \theta = 1 + \tan^2 \theta \quad \text{Pythagorean identity}
\]

\[
\left( \frac{5}{4} \right)^2 = 1 + \tan^2 \theta \quad \text{Substitute } \frac{5}{4} \text{ for } \sec \theta.
\]

\[
\frac{25}{16} = 1 + \tan^2 \theta
\]

\[
\frac{9}{16} = \tan^2 \theta
\]

\[
\pm \frac{3}{4} = \tan \theta
\]

Now find \( \cot \theta \).

\[
\cot \theta = \frac{1}{\tan \theta} \quad \text{Reciprocal identity}
\]

\[
\pm \frac{4}{3}
\]

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Look at table on page 424

Symmetry Identities

\[
\sin (A + 360k) = \sin A \quad \text{and } \cos (A + 360k) = \cos A
\]

\[
\sin (A + 180 (2k - 1)) = -\sin A \quad \text{and } \cos (A + 180 (2k - 1)) = -\cos A
\]

\[
\sin (360k - A) = -\sin A \quad \text{and } \cos (360k - A) = \cos A
\]

\[
\sin (180 (2k - 1) - A) = \sin A \quad \text{and } \cos (180 (2k - 1) - A) = -\cos A
\]

**Example 3**

Express each value as a trigonometric function of an angle in Quadrant I.

a. \( \sin 765^\circ \)

\( 765^\circ \) and \( 45^\circ \) differ by a multiple of \( 360^\circ \).

\[
765^\circ = 45^\circ + 2(360^\circ)
\]

\[
\sin 765^\circ = \sin (45^\circ + 2(360^\circ)) \quad \text{Case 1, with } A = 45^\circ \text{ and } k = 2
\]

\[
= \sin 45^\circ
\]

c. \( \cos 935^\circ \)
Relate 935°

935° = 35° + 5(180°)

935° and 35° differ by an odd multiple of 180°.

Case 2, with A = 35° and k = 3

\[ \cos 935° = \cos (35° + 5(180°)) \]

\[ = - \cos 35° \]

d. \quad \cot \frac{11\pi}{4}

The sum of \( \frac{11\pi}{4} \) and \( \frac{\pi}{4} \), which is \( \frac{12\pi}{4} \) or 3π, is an odd multiple of π.

\[ \frac{11\pi}{4} = 3\pi - \frac{\pi}{4} \]

Case 4 with \( A = \frac{\pi}{4} \) and \( k = 2 \)

\[ \cot \frac{11\pi}{4} = -\cot \frac{\pi}{4} \]

Rewrite using the quotient identity.

\[ = \cot \frac{\pi}{4} \]

Quotient identity

Opposite Angle Identities

\[ \sin (-A) = -\sin A \]

\[ \cos (-A) = \cos A \]

**Example 4**

Simplify \( \cos x \cot x + \sin x \).

\[ \cos x \cot x - \sin x = \cos x \cdot \frac{\cos x}{\sin x} + \sin x \quad \text{Definition of cot } x \]

\[ = \frac{\cos^2 x}{\sin x} + \sin x \]

\[ = \frac{1 - \sin^2 x}{\sin x} + \sin x \quad \text{Pythagorean identity: } \sin^2 x + \cos^2 x = 1 \]

\[ = \frac{1}{\sin x} - \sin x + \sin x \]

\[ = \frac{1}{\sin x} \quad \text{or } \csc x \quad \text{Reciprocal identity} \]

**Example 5**

**PHYSICS** When an object sits at rest, there is no force of friction working on the object. Once a force is applied to slide the object, frictional force is generated. The force of friction opposes the force being applied to the object, and is always equal to this applied force until the object begins to move. When the object begins to slide, applied force becomes greater than the maximum force of
friction, \( F_f^{\text{MAX}} \). The coefficient of static friction \( \mu_s \) is the ratio of the maximal force of friction \( F_f^{\text{MAX}} \) to the normal force \( F_N \), or \( \mu_s = \frac{F_f^{\text{MAX}}}{F_N} \).

### a.
Simplify the equation for coefficient of static friction if \( F_f^{\text{MAX}} = mg \sin \theta \) and \( F_N = mg \cos \theta \).

### b.
Suppose a block is sitting on a flat surface. The surface is raised at one end to form an angle \( \theta = 60^\circ \) with the ground at which point the block begins to slip down the surface. When this happens, the force of friction is at its maximum. What is the coefficient of static friction?

#### a.
\[
\mu_s = \frac{F_f^{\text{MAX}}}{F_N}
\]
\[
\mu_s = \frac{mg \sin \theta}{mg \cos \theta}
\]
\[
\mu_s = \tan \theta
\]

#### b.
\[
\mu_s = \tan 60^\circ
\]
\[
\mu_s = \sqrt{3}
\]
\[
\mu_s \approx 1.732050808
\]

The coefficient of static friction is about 1.73.

Assignment: Pages 427-428

- Day 1: 6, 9, 12, 15, 18, 21, 23, 25, 28, 31, 34, 37
- Day 2: 38, 41, 44, 46-53